## Exercises

E16.17 [1BM] We define the Gamma function $\Gamma : (0,\infty) \to \mathbb{R}$ as

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \, \mathrm{d}t \, .$$

- Show that $\Gamma(x)$ is well defined for $x > 0$ real.
- Show that $\Gamma(x+1) = x\Gamma(x)$ and deduce that $\Gamma(n+1) = n!$ for $n \in \mathbb{N}$.
- Show that $\Gamma(x)$ is analytic.

  *(You can assume that derivatives of $\Gamma$ are $\Gamma^{(n)}(x) = \int_0^\infty (\log t)^n t^{x-1} e^{-t} \, \mathrm{d}t$; those are obtained by derivation under integral sign.)*