Exercises

E15.2 [1BT] Prerequisites:[18F], [1BF]. Let $I \subset \mathbb{R}$ be an open interval. Suppose that $g : I \to \mathbb{R}$ in Riemann integrable on any bounded closed interval contained in I. Fixed $x, y \in \mathbb{R}$ with $x \neq y$, let

$$R(x, y) = \frac{1}{y - x} \int_x^y g(s) \, \mathbb{d}s$$

(with the usual convention that $\int_x^y g(s) \, \mathbb{d}s = - \int_y^x g(s) \, \mathbb{d}s$, so that $R(x, y) = R(y, x)$). If g is monotonic, show that $R(x, y)$ is monotonic in each variable. If g is continuous and $R(x, y)$ is monotonic in each variable, show that g is monotonic.

Solution 1. *[1BV]*