Exercises

E16.2 [1C8] Prerequisites:[1C6].Note:*Darboux properties.*

Let $A \subseteq \mathbb{R}$ be an open set, and suppose that $f : A \to \mathbb{R}$ is differentiable. We want to show that, for each interval $I \subset A$, the image $f'(I)$ is an interval.

So prove this result. For $x, y \in I$ with $x < y$, let's define $a = f'(x), b = f'(y)$. Let's assume for simplicity that $a < b$. For any c with $a < c < b$, there exists $\xi \in I$ with $x < \xi < y$ such that $f'(\xi) = c$.

(Finally, show that this property actually implies that the image $f'(I)$ of an interval I is an interval.)

Solution 1. *[1C9]*