Exercises

16.17 [1DD] Let I be an open interval and $x_0 \in I$, let $f : I \to \mathbb{R}$ be differentiable in I and such that there exists the second derivative f'' in x_0: then show that the limit exists

$$\lim_{t \to 0} \frac{f(x_0 + t) + f(x_0 - t) - 2f(x_0)}{t^2}$$

and that it coincides with $f''(x_0)$.

Find then a simple example of f differentiable in $(-1, 1)$ and such that the second derivative f'' in $x_0 = 0$ does not exist, but the previous limit exists.

Solution 1. *[1DF]*