Exercises

E17.a.4 `Difficulty:*`.Let $n \geq 1$ be an integer. Let I, J be open intervals with $x_0 \in$ [1DJ] $I, y_0 \in J$. Let then be given $g : I \to \mathbb{R}$ and $f : J \to \mathbb{R}$ such that $g(I) \subseteq J$, f, g are $n - 1$ times differentiable in their respective intervals, their $(n - 1)$-th derivative is differentiable in x_0 (resp. y_0) and finally $g(x_0) = y_0$.

Show that the composite function $f \circ g$ is differentiable $n - 1$ times in the interval and its derivative $(n - 1)$-th is differentiable in x_0.

Then write an explicit formula for the nth derivative $(f \circ g)^{(n)}$ in x_0 of the composition of the two functions, (formula that uses derivatives of f and g).

(If you can't find it, read the wikipedia page [61]; or, see this presentation: `https://drive.google.com/drive/folders/` `1746bdJ89ZywciaEqvIMlGZ7kKHWVekhb` *).*

Solution 1. [1DK]