Exercises

16.20 [1DM]Prerequisites:[1DJ],[09N].Show that the function

$$\varphi(x) = \begin{cases} e^{-1/x} & \text{if } \ x > 0 \\ 0 & \text{if } \ x \le 0 \end{cases} \tag{16.20}$$

is of class C^∞, and for $x > 0$

$$\varphi^{(n)}(x) = e^{-1/x} \sum_{m=1}^{n} \binom{n-1}{m-1} \frac{n!}{m!} \frac{(-1)^{m+n}}{x^{m+n}} \quad ,$$

$$\binom{n-1}{m-1} = \frac{(n-1)!}{(n-m)!(m-1)!} \quad .$$

whereas $\varphi^{(n)}(x) = 0$ for each $n \in \mathbb{N}, x \le 0$.

Proceed similarly to

$$\psi(x) = \begin{cases} e^{-1/|x|} & \text{if } \ x \ne 0 \\ 0 & \text{if } \ x = 0 \end{cases} \tag{16.21}$$

again $\psi \in C^\infty$ and $\psi^{(n)}(0) = 0$ for each $n \in \mathbb{N}$; but in this case $\psi(x) = 0 \iff x = 0$.

Solution 1. *[1DN]*