## Exercises

16.22 [1DT] What can you put in place of "???" so that the function

$$g(x) = \begin{cases} ??? & \text{if } \; 0 < x < 1 \,, \\ 1 & \text{if } \; x \geq 1 \,, \\ 0 & \text{if } \; x \leq 0 \,. \end{cases}$$

is $C^\infty$?

More generally, how can two $C^\infty$ functions be connected, so that the whole function is $C^\infty$? Given $f_0, f_1 \in C^\infty$, show [a] that there is a function $f \in C^\infty$ that satisfies

$$\begin{aligned} f(x) &= f_0(x) \quad \text{if} \quad x \leq 0 \quad , \\ f(x) &= f_1(x) \quad \text{if} \quad x \geq 1 \quad . \end{aligned}$$

**Solution 1.** *[1DV]*

[a] Possibly with a simple construction based on example [1DM].