Exercises

E17.31 [1F1] Difficulty:*.Note:*Hadamard's lemma.*

Let $f : \mathbb{R} \to \mathbb{R}$ be a function of class C^∞, and such that $f(0) = 0$. Define, for $x \neq 0$, $g(x) \stackrel{\text{def}}{=} f(x)/x$. Show that g can be prolonged, assigning an appropriate value to $g(0)$, and that the prolonged function is C^∞. What is the relationship between $g^{(n)}(0)$ and $f^{(n+1)}(0)$?

Solution 1. *[1F2]*