Exercises

E17.34 [1F9] Prerequisites:convex functions.Note:*Exercise 1, written exam March 1st, 2010.*

Let's consider the functions $f : \mathbb{R} \to \mathbb{R}$ of class C^∞, such that for every fixed $n \geq 0$, $f^{(n)}(x)$ has constant sign (*i.e.* it is never zero) [a]. We associate to each such function the sequence of signs that are assumed by f, f', f'' ....

What are the possible sequences of signs, and what are the impossible sequences?

(E.g. for $f(x) = e^x$, the associated sequence is $+ + + + + \ldots$, which is therefore a possible sequence.)

See also the exercise [1N7].

[a] We agree that $f^{(0)} = f$.