Definition 16.35. *[1FB]Let $a \in \overline{\mathbb{R}}$ and I be a neighborhood of a. Let $f, g : I \to \mathbb{R}$. We will say that "$f(x) = o(g(x))$ for x tending to a" if* [a]

$$\forall \varepsilon > 0,\ \exists \delta > 0, x \in I \wedge\ |x - a| < \delta \Rightarrow |f(x)| \le \varepsilon |g(x)| \quad .$$

This notation reads like "f is small o of g".
If $g(x) \ne 0$ for $x \ne a$, then equivalently we can write

$$\lim_{x \to a} \frac{f(x)}{g(x)} = 0 \quad .$$

We will say that "$f(x) = O(g(x))$ for x tending to a" if if there is a constant $c > 0$ and a neighborhood J of a for which $\forall x \in J, |f(x)| \le c|g(x)|$.
Again, if $g(x) \ne 0$ for $x \ne a$, then equivalently we can write

$$\limsup_{x \to a} \frac{|f(x)|}{|g(x)|} < \infty \quad ,$$

This notation reads like "f is big O of g".
For further information, and more notations, see [46].
This notation is usually attributed to Landau.

[a] Consider that $J = \{x \in I :\ |x - a| < \delta\}$ is a neighborhood of a.