Example 17.37. *[1FD]We informally state this property.*

$$\text{If } n \geq m \geq 1 \text{ then } o(x^n) + o(x^m) = o(x^m).$$

To prove it, we convert it into a precise statement. First of all, let's rewrite it like this.

$$\text{If } f(x) = o(x^n) \text{ and } g(x) = o(x^m) \text{ then } f(x) + g(x) = o(x^m).$$

So let's prove it. From the hypotheses,

$$\lim_{x \to 0} f(x)x^{-n} = 0 \text{ and } \lim_{x \to 0} g(x)x^{-m} = 0$$

then

$$\lim_{x \to 0} \frac{f(x) + g(x)}{x^m} = \lim_{x \to 0} \frac{f(x)}{x^m} + \lim_{x \to 0} \frac{g(x)}{x^m} = \lim_{x \to 0} x^{n-m} \frac{f(x)}{x^n} + 0 = 0.$$