Example 16.38. [1FF] *We informally state this second property*

$$\text{If } n \geq 1 \text{ then } o\Big(x^n + o(x^n)\Big) = o(x^n).$$

We rewrite it like this.

$$\text{If } f(x) = o(x^n) \text{ and } g(x) = o(x^n + f(x)) \text{ then } g(x) = o(x^n).$$

We note that, for $x \neq 0$ small, $x^n + f(x)$ is not zero, as there is a neighborhood in which $|f(x)| \leq |x^n/2|$. As a hypothesis we have that $\lim_{x\to 0} f(x)x^{-n} = 0$ and $\lim_{x\to 0} g(x)/(x^n + f(x)) = 0$ then

$$\lim_{x\to 0} \frac{g(x)}{x^n} = \lim_{x\to 0} \frac{g(x)}{x^n + f(x)} \frac{x^n + f(x)}{x^n}$$

but

$$\lim_{x\to 0} \frac{g(x)}{x^n + f(x)} = 0$$

while

$$\lim_{x\to 0} \frac{x^n + f(x)}{x^n} = 1 \quad .$$