Exercises

E16.54 [1G8] Let $W \subseteq \mathbb{R}^n$ be an open nonempty set, fix $\overline{x} \in W$. Let then $\psi : W \to \mathbb{R}$ of class C^2. Let $\nabla\psi(\overline{x})$ be the row vector of coordinates $\frac{\partial}{\partial x_k}\psi(\overline{x})$ (which is the gradient of ψ, a special case of the "Jacobian matrix"); we abbreviate it to $D = \nabla\psi(\overline{x})$ for simplicity; let H be the Hessian matrix of components $H_{h,k} = \frac{\partial^2}{\partial x_k x_h}\psi(\overline{x})$; show the validity of Taylor's formula at the second order

$$\psi(\overline{x} + v) = \psi(\overline{x}) + Dv + \frac{1}{2}v^t H v + o(|v|^2)$$

(note that the product Dv is a matrix 1×1 that we identify with a real number, and similarly for $v^t H v$).

[[1G9]]