

Exercises

E16.55 [1GB] Prerequisites: [1G8]. Let $V, W \subseteq \mathbb{R}^n$ be open nonempty sets, and $G : V \rightarrow W$ of class C^2 . Fix $\bar{y} \in V$ and $\bar{x} = G(\bar{y}) \in W$. Suppose that $\psi : W \rightarrow \mathbb{R}$ is of class C^2 ; define $\tilde{\psi} = \psi \circ G$, then compare Taylor's second-order formulas for ψ and $\tilde{\psi}$ (centered in \bar{x} and \bar{y} , respectively). Assuming also that G is a diffeomorphism, verify that

- \bar{x} is a stationary point for ψ if and only if \bar{y} is stationary point for $\tilde{\psi}$,
- and in this case the Hessians of ψ and $\tilde{\psi}$ are similar (i.e. the matrices are equal, up to coordinate changes).

Solution 1. [1GC]