Theorem 16.47. [1GD] *Let* $f : A \subseteq \mathbb{R}^n \to \mathbb{R}$ *be continuous, with* A *open, and let* $\overline{x} = (\overline{x}', \overline{x}_n) \in A$ *be such that* $\partial_{x_n} f$ *exists in a neighborhood of* $\overline{x}$*, is continuous in* $\overline{x}$ *and* $\partial_{x_n} f(\overline{x}) \neq 0$*. Define* $\overline{a} = f(\overline{x})$*.*
There is then a "cylindrical" neighborhood U *of* $\overline{x}$

$$U = U' \times J$$

where

$$U' = B(\overline{x}', \alpha)$$

is the open ball in $\mathbb{R}^{n-1}$ *centered in* $\overline{x}'$ *of radius* $\alpha > 0$*, and*

$$J = (\overline{x}_n - \beta, \overline{x}_n + \beta)$$

with $\beta > 0$*. Inside this neighborhood* $U \cap f^{-1}(\{\overline{a}\})$ *coincides with the graph* $x_n = g(x')$*, with* $g : U' \to J$ *continuous.*
This means that, for every $x = (x', x_n) \in U$*,* $f(x) = \overline{a}$ *if and only if* $x_n = g(x')$*.*
Moreover, if f *is of class* C^k *on* A *for some* $k \in \mathbb{N}^*$*, then* g *is of class* C^k *on* U' *and*

$$\frac{\partial g}{\partial x_i}(x') = -\frac{\dfrac{\partial f}{\partial x_i}(x', g(x'))}{\dfrac{\partial f}{\partial x_n}(x', g(x'))} \qquad \forall x' \in U', \forall i, 1 \leq i \leq n-1 \quad . \tag{16.48}$$