

Exercises

16.48 [1GF] Consider the following C^∞ function of 2 variables

$$f(x, y) = x^3 + y^4 - 1 \quad .$$

Check that $\{f = 0\} = \{(x, y) \in \mathbb{R}^2 : f(x, y) = 0\}$ is not empty; then, for each point of the plane where f vanishes, discuss whether the implicit function theorem can be applied, and therefore if the set $\{f = 0\}$ is locally graph of a C^∞ function. Also study the set $\{f = 0\}$: is it compact? How many connected components are there?

(Please note what is shown in [1H3]).

Solution 1. [1GG]