Exercises

16.48 [1GQ]Let $A \subset \mathbb{R}^3$ be an open set and suppose that $f, g : A \to \mathbb{R}$ is differentiable, and such that in $p_0 = (x_0, y_0, z_0) \in A$ we have that $\nabla f(p_0), \nabla g(p_0)$ are linearly independent and $f(p_0) = g(p_0) = 0$: show that the set $E = \{f = 0, g = 0\}$ is a curve in a neighborhood of p_0.

(Hint: consider that the vector product $w = \nabla f(p_0) \times \nabla g(p_0)$ is nonzero if and only if the vectors are linearly independent — in fact it is formed by the determinants of the minors of the Jacobian matrix. Assuming without loss of generality that $w_3 \neq 0$, show that E is locally the graph of a function $(x, y) = \gamma(z)$.)

Solution 1. *[1GR]*