

Exercises

16.48 [1GS] *Note: Written exam, July 4th 2018.* The figure 6 shows the set $E = \{(x, y) : ye^x + xe^y = 1\}$.

Properly prove the following properties:

- (i) at every point $(x_0, y_0) \in E$ the assumptions of the implicit function theorem are satisfied;
- (ii) $E \cap \{(x, y) : x > 0\}$ coincides with the graph, in the form $y = f(x)$, of a single function f defined on $(0, +\infty)$;
- (iii) E is connected;
- (iv) $\lim_{x \rightarrow +\infty} f(x) = 0$.
- (v) Show (at least intuitively) that $x_0 > 0$ exists with the property that f is decreasing for $0 < x < x_0$, increasing for $x > x_0$.

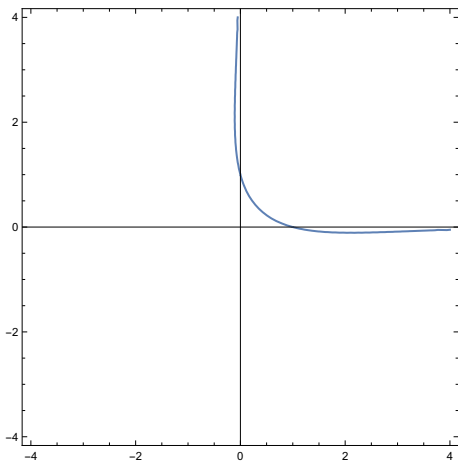


Figure 6: Figure for exercise E16.48.

Solution 1. [1GV]