Exercises

16.54 [1GZ] In the same assumptions as the previous theorem [1GD], show that there exist $\varepsilon > 0$ and a continuous function $\tilde{g} : V \to \mathbb{R}$ where $I = (\overline{a} - \varepsilon, \overline{a} + \varepsilon)$ and $V = U' \times I$ is open in $\mathbb{R}^n$, such that

$$\forall (x', a) \in V \quad , \quad (x', \tilde{g}(x', a)) \in U \quad \text{e} \quad f\big(x', \tilde{g}(x', a)\big) = a \quad . \tag{16.54}$$

Vice versa if $x \in U$ and $a = f(x)$ and $a \in I$ then $x_n = \tilde{g}(x', a)$.

Note that the previous relation means that, for each fixed $x' \in U'$, the function $\tilde{g}(x', \cdot)$ is the inverse of the function $f(x', \cdot)$ (when defined on appropriate open intervals).

So, moreover, the function $\tilde{g}$ is always differentiable with respect to a, and the partial derivative is

$$\frac{\partial}{\partial a}\tilde{g}(x', a) = \frac{1}{\frac{\partial}{\partial x_n} f(x', \tilde{g}(x', a))} \quad .$$

The other derivatives instead (obviously) are as in the theorem [1GD].

The regularity of $\tilde{g}$ is the same as g: if f is Lipschitz then $\tilde{g}$ is Lipschitz; if $f \in C^k(U)$ then $\tilde{g} \in C^k(V)$.

Solution 1. *[1H0]*