

Exercises

E17.d.14 [1H3] Prerequisites: [0JV], [0JY], [0T5], [0T7], [1DT], [1GD], [1P1] and [1PG].

Difficulty:**.

For this exercise we need definitions and results presented in the Chapter [1NT].

Let $r \geq 1$ integer, or $r = \infty$. Let $F : \mathbb{R}^2 \rightarrow \mathbb{R}$ of class C^r , and such that $\nabla F \neq 0$ at every point $F = 0$.

We know, from [0JV], that $\{F = 0\}$ is the disjoint union of connected components, and from [0JY] that every connected component is a closed.

Show that, for every connected component K , there is an open set $A \supseteq K$ such that $K = A \cap \{F = 0\}$, and that therefore there are at most countably many connected components.

Show that each connected component is the support of a simple immersed curve of class C^r , of one of the following two types:

- the curve is closed, or
- The curve $\gamma : \mathbb{R} \rightarrow \mathbb{R}^2$ is not closed and is unbounded (i.e. $\lim_{t \rightarrow \pm\infty} |\gamma(t)| = \infty$).

The first case occurs if and only if the connected component is a compact.

Solution 1. [1H4]