**Exercises**

16.59 [1HD] In the same hypotheses, we see a "vice versa". Let $f, \varphi : A \to \mathbb{R}$ be of class $C^2$ in the open set $A$, and let $\overline{x} \in E_a$ and $\lambda \in \mathbb{R}$ be such that $\nabla f(\overline{x}) + \lambda \nabla \varphi(\overline{x}) = 0$; suppose that

$$\forall v, v \cdot \nabla \varphi(x) = 0 \Longrightarrow v \cdot Hv > 0$$

where

$$h(x) = f(x) + \lambda \varphi(x)$$

and $H$ is the Hessian matrix of $h$ in $\overline{x}$. Show that $\overline{x}$ is a local minimum point for $f$ bound to $E_a$.

**Solution 1.** [1HF]