Definition 18.3. [1HR] *Let (X_1, d_1) and (X_2, d_2) be metric spaces. Let $\mathcal{F}$ be a family of functions $f : X_1 \to X_2$, we will say that it is an **equicontinuous family** if one of these equivalent properties holds.*

- $\forall \varepsilon > 0\ \exists \delta > 0\ \forall f \in \mathcal{F}$
$$\forall x, y \in X_1,\ d_1(x, y) \le \delta \Rightarrow d_2(f(x), f(y)) \le \varepsilon \quad .$$
- *There exists a a fixed monotonically weakly increasing function $\omega : [0, \infty) \to [0, \infty]$, for which $\lim_{t \to 0+} \omega(t) = \omega(0) = 0$ (ω is called* "continuity modulus" [a]*) such that*
$$\forall f \in \mathcal{F},\ \forall x, y \in X_1,\ d_2(f(x), f(y)) \le \omega\big(d_1(x, y)\big) \quad . \tag{18.4}$$
- *There exists a fixed continuous function $\omega : [0, \infty) \to [0, \infty]$ with $\omega(0) = 0$ such that* (18.4) *holds.*

(The result [150] can be useful to prove equivalence of the last two clauses.)

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[a] See also [156], regarding the notion of "continuity modulus".