

Exercises

E17.3 [1HS] Note: *This result is known as "Dini's lemma".*

Let (X, d) be a metric space, let $I \subset X$ be a compact set, and suppose that $f, f_n : I \rightarrow \mathbb{R}$ are continuous and such that $f_n(x) \searrow_n f(x)$ pointwise (i.e. for every $x \in I$ and n we have $f(x) \leq f_{n+1}(x) \leq f_n(x)$ and $\lim_n f_n(x) = f(x)$). Show that $f_n \rightarrow f$ uniformly.

Solution 1. [1HT]

Solution 2. [1HV]

In following exercises we will see that, if even one of the hypotheses fails, then there are counterexamples.