Exercises

E17.9 [1J3] Let $I \subset \mathbb{R}$ be an interval. Which of these classes $\mathcal{F}$ of functions $f : I \to \mathbb{R}$ are closed for uniform convergence? Which are closed for pointwise convergence?

1. The continuous and monotonic (weakly) increasing functions on $I = [0, 1]$.

   Solution 1. *[1J4]*

2. The convex functions on $I = [0, 1]$.

   Solution 2. *[1J5]*

3. Given $\omega : [0, \infty) \to [0, \infty)$ a fixed continuous function with $\omega(0) = 0$ (which is called *"continuity modulus"*), and

   $$\mathcal{F} = \{f : [0, 1] \to \mathbb{R} \ : \ \forall x, y, |f(x) - f(y)| \le \omega(|x - y|)\}$$

   (this is called *a family of equicontinuous functions*, as explained in the definition [1HR].)

   Solution 3. *[1J6]*

4. Given $N \ge 0$ fixed, the family of all polynomials of degree less than or equal to N, seen as functions $f : [0, 1] \to \mathbb{R}$.

   Solution 4. *[1J7]*

5. The regulated functions on $I = [0, 1]$. [a]

   Solution 5. *[1J9]*

6. The uniformly continuous and bounded functions on $I = \mathbb{R}$.

   Solution 6. *[1JB]*

7. The Hoelder functions on $I = [0, 1]$, i.e.

   $$\left\{f : [0, 1] \to \mathbb{R} \,\middle|\, \exists b > 0, \exists \alpha \in (0, 1] \ \forall x, y \in [0, 1], |f(x) - f$$

   Solution 7. *[1JC]*

8. The Riemann integrable functions on $I = [0, 1]$.

   Solution 8. *[1JF]*

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[a] *Regulated* functions $f : I \to \mathbb{R}$ are the functions that, at each point, have finite left limit, and finite right limit. See Section [2CT].