Exercises

E17.8 [1JS] Let $I \subset \mathbb{R}$ be an open set, and let $\hat{x}$ be an accumulation point for I [a] , Let $f_m : I \to \mathbb{R}$ be a sequence of bounded functions that converge uniformly to $f : I \to \mathbb{R}$ when $m \to \infty$. Suppose that, for every m, there exists the limit $\lim_{x\to\hat{x}} f_m(x)$, then

$$\lim_{m\to\infty} \lim_{x\to\hat{x}} f_m(x) = \lim_{x\to\hat{x}} \lim_{m\to\infty} f_m(x)$$

in the sense that if one of the two limits exists, then the other also exists, and they are equal. (The above result also applies to right limits or left limits.)

Show with a simple example that, if the limit is not uniform, then the previous equality does not hold.

Solution 1. *[1JT]*

(See also the exercise [0CX]).

[a] Including also the case where I is not upper bounded, and $\hat{x} = +\infty$; or the case where I is not lower bounded and $\hat{x} = -\infty$.