Exercises

E17.8 [1JV] Let $I \subset \mathbb{R}$ be a compact interval, let $f_n, f : I \to \mathbb{R}$ be continuous. Show that the following two facts are equivalent.

a. For every $x \in X$ and for every sequence $(x_n)_n \subset I$ for which $x_n \to_n x$, we have $\lim_{n\to\infty} f_n(x_n) = f(x)$;

b. $f_n \to_n f$ uniformly on I.

Then find an example where $I = [0, 1)$, the first point holds, but f_n does not tend uniformly to f.

Solution 1. *[1JW]*