

Exercises

E18.a.2 [1JX] Prerequisites: [1J3] subpoint [6], [1JN]. Let $I \subseteq \mathbb{R}$ be a subset. Let X be the set of functions $f : I \rightarrow \mathbb{R}$ bounded and uniformly continuous. We equip X with distance $d_\infty(f, g) = \|f - g\|_\infty$. Show that the metric space (X, d_∞) is complete.

Solution 1. [1JY]

In particular, X is a closed vector subspace of the space $C_b(I)$ of continuous and bounded functions.

[[1JZ]]