## Exercises

E18.7 [1KM] Let $b \in \mathbb{R}$, $n \in \mathbb{N}$. Assuming that $f(t) = \sum_{k=0}^{\infty} a_k t^k$ with radius of convergence $r$ positive and $t \in (-r, r)$, determine the coefficients $a_k$ so as to satisfy the following differential equations.,

1. $f'(t) = f(t)$ and $f(0) = b$,
2. $f'(t) = t^2 f(t)$ and $f(0) = b$,
3. $f''(t) = t^2 f(t)$ and $f(0) = b$, $f'(0) = 0$,
4. $tf''(t) + f'(t) + tf(t) = 0$ and $f(0) = b$, $f'(0) = 0$,
5. $t^2 f''(t) + tf'(t) + (t^2 - m^2) f(t) = 0$ $m \geq 2$ integer, $f(0) = f'(0) = \ldots f^{(m-1)} = 0$, and $f^{(m)} = b$.

(The last two are called *Bessel equations*). [1KN]

**Solution 1.** *[1KP]*