Exercises

E18.1 [1KM] Let $b \in \mathbb{R}$, $n \in \mathbb{N}$. Assuming that $f(t) = \sum_{k=0}^{\infty} a_k t^k$ with radius of convergence r positive and $t \in (-r, r)$, determine the coefficients a_k so as to satisfy the following differential equations.,

(a) $f'(t) = f(t)$ and $f(0) = b$,

(b) $f'(t) = t^2 f(t)$ and $f(0) = b$,

(c) $f''(t) = t^2 f(t)$ and $f(0) = b, f'(0) = 0$,

(d) $tf''(t) + f'(t) + tf(t) = 0$ and $f(0) = b, f'(0) = 0$,

(e) $t^2 f''(t) + tf'(t) + (t^2 - m^2)f(t) = 0$ $m \geq 2$ integer, $f(0) = f'(0) = \ldots f^{(m-1)} = 0$, and $f^{(m)} = b$.

(The last two are called *Bessel equations*). [1KN]

Solution 1. *[1KP]*