Exercises

E18.7 [1KQ]Prerequisites:[1K9].Consider power series

$$f(x) = \sum_{n=0}^{\infty} a_n x^n \quad , \quad g(x) = \sum_{m=0}^{\infty} b_m x^m \ ,$$

with non-zero radius of convergence, respectively r_f and r_g.

Show that the product function $h(x) = f(x)g(x)$ can be expressed in power series

$$h(x) = \sum_{k=0}^{\infty} c_k x^k$$

where

$$c_k = \sum_{j=0}^{k} a_j b_{k-j} \ ;$$

with radius of convergence $r_h \geq \min\{r_f, r_g\}$. (Note the similarity with Cauchy's product, discussed in section [0CN])

Can it happen that $r_h > \min\{r_f, r_g\}$?

Solution 1. *[1KR]*