Exercises

E18.7 Prerequisites: [1K9], [1DJ]. Difficulty: *.

Consider the power series

$$f(x) = \sum_{n=0}^{\infty} a_n x^n \ , \ \ g(x) = \sum_{m=0}^{\infty} b_m x^m \ ,$$

with non-zero radius of convergence, respectively r_f and r_g. Suppose $g(0) = 0 = b_0$. Let $I_f, I_g \subset \mathbb{C}$ be disks centered in zero with radii less than r_f and r_g, respectively: the previous series therefore define functions $f : I_f \to \mathbb{C}$ and $g : I_g \to \mathbb{C}$. Up to shrinking I_g, we assume that $g(I_g) \subset I_f$.

Show that the composite function $h = f \circ g : I_g \to \mathbb{C}$ can be expressed as a power series $h(x) = \sum_{k=0}^{\infty} c_k x^k$ (with radius of convergence at least r_g). Show how coefficients c_k can be computed from coefficients a_k, b_k.

Solution 1. [1KW]