Exercises

18.13 [1MD] We define the functions *hyperbolic cosine* [a]

$$\cosh y = \frac{e^y + e^{-y}}{2}$$

and *hyperbolic sine*

$$\sinh y = \frac{e^y - e^{-y}}{2} .$$

- Verify that
$$(\cosh x)^2 - (\sinh x)^2 = 1$$
(which justifies the name of "hyperbolic").
- Prove the validity of these power series expansion
$$\cosh(x) = 1 + \frac{1}{2}x^2 + \frac{1}{4!}x^4 + \frac{1}{6!}x^6 + \ldots$$
$$\sinh(x) = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \frac{1}{7!}x^7 + \ldots$$
- Check that
$$\cosh' = \sinh \ , \ \ \sinh' = \cosh$$
- Check the formulas
$$\sinh(x + y) = \cosh x \sinh y + \cosh y \sinh x$$
$$\cosh(x + y) = \cosh x \cosh y + \sinh y \sinh x \ .$$

[a] See Wikipedia page "Derivazione delle funzioni iperboliche" [28] which explains in what sense y is an "angle".