Exercises

18.20 [1MW] Difficulty:*. In the general case (when we do not know if A, B commute) we proceed as follows. Let's define $[A, B] = AB - BA$.

- Setting $B_0 = B$ and $B_{n+1} = [A, B_n]$ you have

$$\begin{aligned} B_n &= A^n B - nA^{n-1}BA + \frac{n(n-1)}{2}A^{n-2}BA^2 + \cdots + (-1) \\ &= \sum_{k=0}^{n} (-1)^k \binom{n}{k} A^{n-k}BA^k \ ; \end{aligned}$$

- let's define now $Z = Z(A, B)$

$$Z \stackrel{\text{def}}{=} \sum_{n=0}^{\infty} \frac{B_n}{n!} \ , \tag{18.20}$$

(note that Z is linear in B): prove that the above series converges, and that

$$\exp(A)B\exp(-A) = Z \ ; \tag{18.21}$$

- from this finally it is shown that

$$\exp(A)\exp(B)\exp(-A) = \exp(Z) \ .$$

(These formulas can be seen as consequences of the Baker–Campbell–Hausdorff formula [39]).

Solution 1. *[1MX]*