Exercises

E19.1 [1NG] Let $f : \mathbb{R} \to \mathbb{R}$ be a C^∞ class function; fix $x_0 \in \mathbb{R}$ and define

$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

using the Taylor series; suppose g has radius of convergence $R > 0$: So $g : J \to \mathbb{R}$ is a well-defined function, where $J = (x_0 - R, x_0 + R)$. Can it happen that $f(x) \neq g(x)$ for a point $x \in J$?

And if f is analytic? [a]

Solution 1. *[1NH]*

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[a] By "analytic" we mean: fixed x_0 there is a series $h(x) = \sum_{n=0}^{\infty} a_n(x - x_0)^n$ with non-zero radius of convergence such that $f = h$ in an open neighborhood of x_0 (neighborhood contained in the convergence disk) .