

Exercises

E21.8 [1P1] Prerequisites: [1P0]. Difficulty: *.

Fix a curve $\gamma : I \rightarrow \mathbb{R}^n$. We define in the following $\hat{I} = \{t \in \mathbb{R} : -t \in I\}$ and $\hat{\gamma} : \hat{I} \rightarrow \mathbb{R}^n$ via $\hat{\gamma}(t) = \gamma(-t)$.

We want to show that, in certain hypotheses, two curves have the same support if and only if they are equivalent.

- Let $\gamma, \delta : [0, 1] \rightarrow \mathbb{R}^n$ be simple curves, but not closed, and with the same support. Show that if $\gamma(0) = \delta(t)$ then $t = 0$ or $t = 1$. In case $\gamma(0) = \delta(0)$, show that $\gamma \sim \delta$. If instead $\gamma(0) = \delta(1)$ then $\hat{\gamma} \sim \delta$.
- Let $\gamma, \delta : [0, 1] \rightarrow \mathbb{R}^n$ be simple immersed curves, but not closed, and with the same support, and let $\gamma(0) = \delta(0)$: show that $\gamma \approx \delta$. If instead $\gamma(0) = \delta(1)$ then $\hat{\gamma} \approx \delta$.

(For the case of closed curves see [1PT])

Solution 1. [1P2]