

Exercises

E20.4 [1P7] Note: Nice formula taken from [56].

Let $S = S(0, 1) \subseteq \mathbb{R}^n$ be the unit sphere $S = \{x : |x| = 1\}$. Let $v, w \in S$ with $v \neq w$ and $v \neq -w$; let $T = \arccos(v \cdot w)$ so that $T \in (0, \pi)$; then the geodesic (that is, the arc-parameterized minimal length curve) $\gamma(t) : [0, T] \rightarrow S$ connecting v to w inside S is

$$\gamma(t) = \frac{\sin(T-t)}{\sin(T)}v + \frac{\sin(t)}{\sin(T)}w \quad ,$$

and its length is T .

(You may assume that, when $v \cdot w = 0$ that is $T = \pi/2$, then the geodesic is $\gamma(t) = v \cos(t) + w \sin(t)$).

Solution 1. [1P8]