Exercises E21.a.9 [1PQ]Prerequisites: [1NX], [1PG]. Let  $\gamma$ ,  $\delta$  curves be closed and immersed, but seen as maps defined on  $\mathbb{R}$  and  $C^1$  and periodic. with

periods 1.

Compare it with the relation  $\approx$ .

Let's see a new relation: you have  $\gamma \approx_f \delta$  if there is an increasing diffeomorphism  $\varphi: \mathbb{R} \to \mathbb{R}$  such that  $\varphi(t+1) = \varphi(t) + 1$  for every

 $t \in \mathbb{R}$  and for which  $\gamma = \delta \circ \varphi$ 

Show that this is an equivalence relation.