

Exercises

E21.1 [1PX] Note: exercise 4, written exam 20 June 2017.

Let F be a continuous vector field on $\mathbb{R}^n \setminus \{0\}$, such that, for every $x \neq 0$, $F(x)$ is a scalar multiple of x . For $r > 0$, we denote with S_r the sphere of radius r centered in 0.

- Prove that, for each regular arc γ with support contained in a sphere S_r , we have $\int_{\gamma} F = 0$.
- Prove that, if such a field F is conservative, then $|F(x)|$ is constant on every sphere S_r , and therefore that $F(x) = x\rho(|x|)$ with $\rho : \mathbb{R}^n \setminus \{0\} \rightarrow \mathbb{R}$ continuous.

Solution 1. [1PY]