

## §23 Ordinary Differential equations

[1QB]

To solve the following exercises, it is important to know some fundamental results, such as: the existence and uniqueness theorem<sup>†115</sup>, Gronwall's Lemma; and in general some methods to analyze, solve and qualitative study Ordinary Differential Equations (abbreviated ODE). These may be found *e.g.* in [?, ?, ?].

### Exercises

E23.1 [1QC]

E23.2 [1QH]

E23.3 [1QK]

E23.4 [1QN]

### §23.a Autonomous problems

#### Exercises

E23.a.1 [1QR]

E23.a.2 [1QV]

E23.a.3 [1QX]

E23.a.4 [1QZ]

### §23.b Resolution

#### Exercises

E23.b.1 [1R1]

E23.b.2 [1R4]

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<sup>†115</sup>*A.k.a.* Picard–Lindelöf theorem, or Cauchy–Lipschitz theorem.

**§23.c Qualitative discussions**

[ [1R6]

For the following exercises the following simple comparison lemma may be useful.

**Lemma 23.c.1.** [1R7]

(There are much more refined versions of this lemma, see for example in section 8.6 in the course notes [?]).

**Exercises**

E23.c.2 [1R8]

E23.c.3 [1RD]

E23.c.4 [1RK]

E23.c.5 [1RQ]

**QuasiEsercizio 1.** [1RR]**QuasiEsercizio 2.** [1RS]

## §23.d Envelope

Given a family of planar curves, we want to define the *envelope curve*. Let's see two possible definitions.

**Definition 23.d.1** (Curve Envelope). [23Y]

**Remark 23.d.2.** [240]

[1RT]

We want to see that the two previous definitions are equivalent in this sense.

### Exercises

E23.d.3 [1RV]

E23.d.4 [1RY]

E23.d.5 [1S1]

E23.d.6 [1S4]

E23.d.7 [1S7]

E23.d.8 [1S9]

**§23.e Linear equations (with constant coefficients)****Definition 23.e.1.** [23Z]**Exercises**

E23.e.2 [1SC]

E23.e.3 [1SD]

E23.e.4 [1SF]

E23.e.5 [1SH]

E23.e.6 [1SK]

E23.e.7 [1SN]

E23.e.8 [1SP]

E23.e.9 [1SS]

**§23.f Matrix equations**

To solve the following exercises you need to know the elementary properties of the exponential of matrices, see section [\[2D8\]](#).

**Exercises**E23.f.1 [\[1S1\]](#)E23.f.2 [\[1S2\]](#)E23.f.3 [\[1T1\]](#)E23.f.4 [\[1T3\]](#)E23.f.5 [\[1T6\]](#)