Exercises

E23.3 [1QH] Prerequisites: [1CB].

Let $I \subseteq \mathbb{R}$ be an open interval.

Let $F : I \times \mathbb{R} \to (0, \infty)$ be a positive continuous function, and let $f : I \to \mathbb{R}$ be a differentiable function that solves the differential equation

$$(f'(x))^2 = F(x, f(x)) \quad .$$

Prove that x is, either always increasing, in which case $f'(x) = \sqrt{F(x, f(x))}$ for every x, or it is always decreasing, in which case $f'(x) = -\sqrt{F(x, f(x))}$; therefore f is of class C^1.

Solution 1. [1QJ]