Exercises

E23.4 [1QK] Prerequisites: [1QH].

Describe all the differentiable functions $f : \mathbb{R} \to \mathbb{R}$ that solve

$$\forall x\ ,\ (f'(x))^2 + (f(x))^2 = 1\ .$$

Show that if $-1 < f(x) < 1$ for $x \in I$ open interval, then f is a sine arc, for $x \in I$.

Show that all solutions are C^1, and that they are piecewise C^∞.

Note that $f \equiv 1$ and $f \equiv -1$ are envelopes of the other solutions, as explained in the section [1QB].

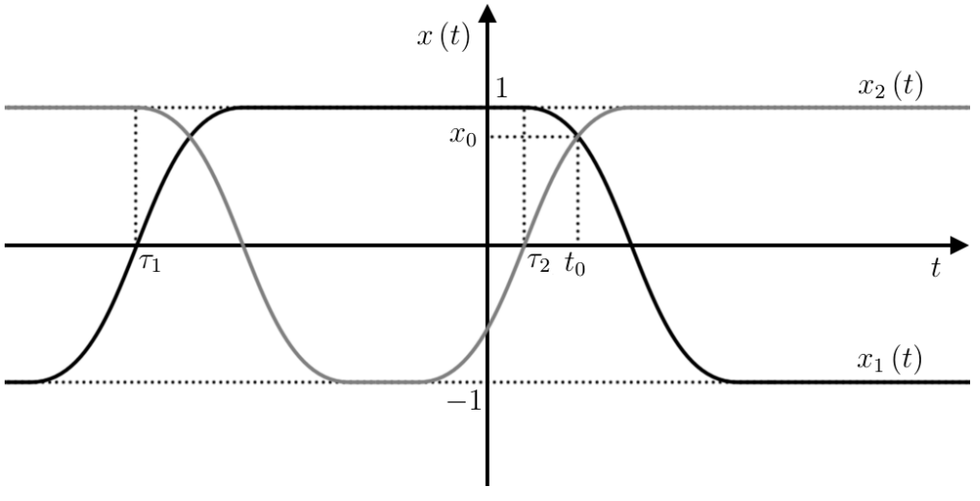

Figure 9: Figure for [1QK]

Solution 1. *[1QM]*