Exercises

E23.8 [1QZ] Prerequisites: [1QV]. Let us fix $\alpha > 1$, and consider again

$$\begin{cases} x'(t) = |x(t)|^\alpha \ , \\ x(0) = 1 \end{cases}$$

We have seen in [1QV] that this ODE admits a maximal solution $x : I_\alpha \to \mathbb{R}$. Fixed $t \in \mathbb{R}$, show that $t \in I_\alpha$ for $\alpha > 1$ close to 1, and that $\lim_{\alpha \to 1+} x(t) = e^t$.

Note that e^t is the only solution of $x'(t) = |x(t)|$ with $x(0) = 1$.

Solution 1. [1R0]