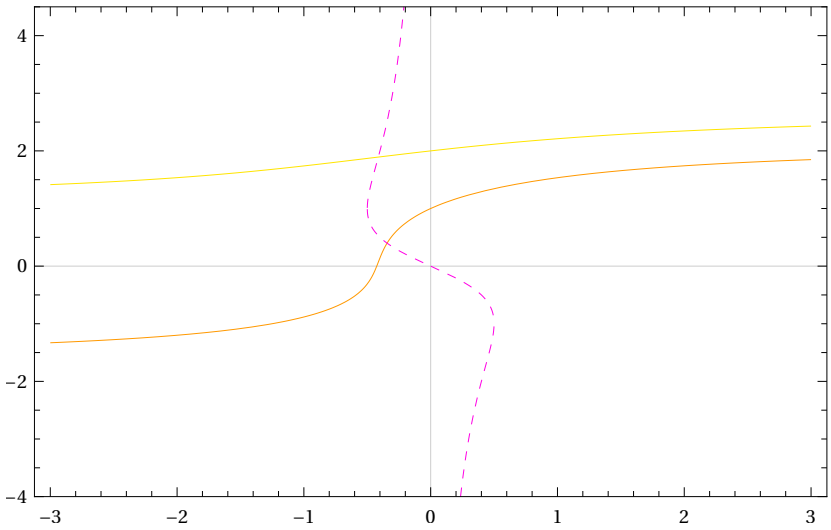

In dashed purple the line of inflections. In yellow the solutions with initial data $y(0) = 1$ and $y(0) = 2$.

Figure 8: Exercise 22.16.

Exercises

E22.16 Considering the Cauchy problem [1RD]

$$\begin{cases} y'(x) = \frac{1}{y(x)^2 + x^2} \\ y(0) = 1 \end{cases}$$

show that there is only one global solution $y : \mathbb{R} \to \mathbb{R}$, that y is bounded, and the limits $\lim_{x\to\infty} y(x)$, $\lim_{x\to-\infty} y(x)$ exist and are finite.

Solution 1. *[1RG]*