



Figure 11: Exercise 23.17. In purple the line of inflections. In yellow the solutions with initial data $y(0) = 1$ and $y(0) = 2$.

Exercises

E23.17 Considering the Cauchy problem

$$\begin{cases} y'(x) = \frac{1}{y(x)^2 + x^2} \\ y(0) = 1 \end{cases}$$

[1RD]

show that there is only one global solution $y : \mathbb{R} \rightarrow \mathbb{R}$, that y is bounded, and the limits $\lim_{x \rightarrow \infty} y(x)$, $\lim_{x \rightarrow -\infty} y(x)$ exist and are finite.

Solution 1. [1RG]