

Exercises

E23.22 [1RV] Let's start with the first definition. Suppose we can apply the Implicit Function Theorem to the locus

$$E_a = \{(x, a) : F(x, y, a) = 0\} \quad ;$$

Precisely, suppose that at a point $(\bar{x}, \bar{y}, \bar{a})$ we have that $\frac{\partial F}{\partial y} \neq 0$.

To this we also add the hypothesis $\frac{\partial^2 F}{\partial a a} \neq 0$. Fixed a , you can express E_a locally as a graph $y = f(x, a) = f_a(x)$. We also use the hypothesis $\frac{\partial^2 F}{\partial a a} \neq 0$ to express locally $\frac{\partial F}{\partial a} = 0$ as a graph $a = \Phi(x, y)$. Defining $G(x, y) \stackrel{\text{def}}{=} F(x, y, \Phi(x, y))$, show that $G = 0$ can be represented as $y = g(x)$. Finally, show that g is the envelope of the curves f_a .

Solution 1. [1RW]