Exercises

E23.33 [1SK] Prerequisites: [19M], [1SF].

Fix $\theta, \tau \in \mathbb{C}$ with $\theta \neq \tau$, $q(x)$ a polynomial, and $k \in \mathbb{N}$. Let's define $p(x) = (x - \theta)^k$. Show that

$$p(D)f(x) = e^{\tau x} q(x)$$

if and only if

$$f(x) = e^{\theta x} r(x) + e^{\tau x} \tilde{q}(x)\,,$$

with r polynomial of degree at most $k - 1$ and $\tilde{q}$ polynomial of the same degree as q.

Solution 1. *[1SM]*