Exercises

23.33 [1T3] Prerequisites: [1T1], E23.33. Note: *Abel's identity*.

Let be given $C \in \mathbb{C}^{n\times n}$, $A : \mathbb{R} \to \mathbb{C}^{n\times n}$ continuous, and the solution $Y(t)$ of the ODE

$$\frac{d}{dt}Y(t) = A(t)Y(t) \quad , \quad Y(0) = C$$

(which has been studied in [1T1]). Set $a(t) = \mathrm{tr}(A(t))$, show that

$$\det(Y(t)) = \det(C)e^{\int_0^t a(\tau)\,\mathbb{d}\tau} \quad .$$

If C is invertible, it follows that $Y(t)$ is always invertible.

Solution 1. *[1T4]*

[[1T5]]