Exercises

E24.4 [1TG] Note: *adapted from the written exam, April 9th, 2011*.

Let $f : [0,\infty) \to \mathbb{R}$ be a continuous function such that

$$\lim_{x\to+\infty} f(x)/x = +\infty \quad .$$

- Fixed $a < f(0)$, let M_a be the set of $m \in \mathbb{R}$ such that the line $y = mx + a$ intersects the graph $y = f(x)$ of the function f at least in one point: show that M_a admits minimum $\hat{m} = \hat{m}(a)$;
- show that $\hat{m}$ depends continuously on a, [a]
- and that $\hat{m}(a)$ is monotonic strictly decreasing.
- If f is differentiable, show that the line $y = \hat{m}(a)x + a$ is tangent to the graph at all points where it encounters it.
- Suppose further that f is of class C^2 and that $f''(x) > 0 \forall x > 0$ [b]. Show that there is only one point x where the line $y = \hat{m}(a)x + a$ meets the graph $y = f(x)$; name it $\hat{x} = \hat{x}(a)$;
- and show that the functions $a \mapsto \hat{x}(a)$ and $a \mapsto \hat{m}(a)$ are differentiable.

Solution 1. *[1TH]*

---

[a] Tip: Rethink the exercise [14W].

[b] Use the previous exercise [1TD]!