Exercises

E24.5 [1TJ] `Topics:osculating circle.` `Note:`*adapted from the written exam, April 9th 2011.*

Let $f : \mathbb{R} \to \mathbb{R}$ be twice differentiable in 0, with $f(0) = 0$ and $f''(0) \neq 0$. Prove that there is an unique point $P = (a, b)$ in the plane and an unique constant $r > 0$, such that

$$d\big(P, (x, f(x))\big) = r + o(x^2),$$

determining a, b, r as a function of $f'(0), f''(0)$. Here $d(P, Q)$ is the Euclidean distance between two points P, Q in the plane.

Hint. First, study the case in which also $f'(0) = 0$.

(The graph of the function f is a curve in the plane; by hypothesis this curve passes through the origin. In this exercise we have determined the circle, of radius r and center P, which best approximates the curve near the origin. This circle is called the "osculating circle", and its radius is called the "radius of curvature", and the inverse of the radius is the "curvature" of the curve at the origin.)

Solution 1. *[1TK]*