Exercises

E24.1 [1TX] Note: *Dirichlet criterion for integrals.*

Let $f, g : [0, \infty) \to \mathbb{R}$ be continuous, where f is positive and monotonic decreasing with $\lim_{x \to \infty} f(x) = 0$, while

$$\sup_{x > 0} | \int_0^x g(t) \, dt | < \infty \quad .$$

Then prove that

$$\lim_{x \to \infty} \int_0^x f(t) g(t) \, dt$$

converges.