

Exercises

E24.15 [1V2] Topics: matrix, determinant. Difficulty: *.

Prove Jacobi's formula:

$$\frac{d}{da_{i,j}} \det(A) = C_{i,j} \quad ,$$

where $a_{i,j}$ is the element of A in row i and column j , and C is the matrix of cofactors of A , which is the transpose of the adjoint matrix $\text{adj}(A)$. Consequently, if $F : \mathbb{R} \rightarrow \mathbb{C}^{n \times n}$ is differentiable, then

$$\frac{d}{dt} \det F(t) = \text{tr} \left(\text{adj}(F(t)) \frac{dF(t)}{dt} \right)$$

where $\text{tr}(X)$ is the trace of X .

Hint: use Laplace's expansion for the determinant.

Solution 1. [1V3]