

## Exercises

24.17 [1VC] Difficulty:\*. Note: exercise 3, written exam, June 30th, 2017.

Consider the problem

$$\begin{cases} y'(x) = y(x^2) \\ y(0) = 1 \end{cases}$$

(this is not a Cauchy problem).

- Show that, for every  $r < 1$ , there is only one solution defined on  $I = (-r, r)$ , and deduce that the same is true for  $r = 1$ .
- Show that the solution is representable as the sum of a power series centered in 0 and converging on the interval  $[-1, 1]$ .

**Solution 1.** [1VD]